# Speed-Power Models – A Bayesian Approach

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**Abstract**

*Hull and propeller performance, especially after the establishment of ISO 19030, has been a major concern for the vessel operators and their energy efficiency departments. However, although latest development in this area is mature, the establishment of vessel models is still an area with a high level of uncertainty. In this paper a gaussian process regression is applied to in-service data to model both speed-power curves and the underlying uncertainty. Furthermore, the mean values of the posterior distribution of the model are used as a speed-power reference curve and the results are compared to widely used speed-power reference curves from speed trials. Finally, the same process is used to model fuel oil consumption and discuss the results.*

## 1. Introduction

Within the last years, energy efficiency in the marine industry was one of the main focus areas. There was big focus on how to design more efficient vessels with significantly improved Energy Efficiency Design Index (EEDI), the overall budget for energy efficiency solutions was drastically increased and several new operational measures were introduced (i.e. slow steaming, performance monitoring, trim optimization, etc.) in order to reduce fuel consumption and $CO_2$ emissions. In addition, regulators all over the world are focused on several environmental protection measures and marine industry is high on their agenda. Therefore, several regulations/restrictions were imposed which hopefully will result into "greener" vessels.

Although mainly driven by regulators, but not only, the marine industry quickly adopted to these changes and International Maritime Organization (IMO) introduced in 2018 some long-term strategies in an attempt to drastically change the environmental footprint of marine industry. As per *IMO (2018)*, the marine industry should reduce $CO_2$ emissions per transport work, as an average across international shipping, by at least 40% by 2030 and pursuing efforts towards 70% by 2050, or at least 50% by 2050 and pursuing efforts towards phasing them out compared to 2008. In an attempt to meet these optimistic targets, operators are more and more focused on how to improve operational efficiency of their fleet. One effective way to do that, is to monitor their vessel's performance and apply corrective actions/ measures when vessel is not performing. So the question is, when does the vessel not perform?

Almost all available methods to answer this question follow the same principles. A model of the vessel is created which in theory describes the vessel's optimal performance. Through monitoring and data logged from sensors onboard, the deviation from the model is recorded (usually over time). If the deviation is high, the reason for this deviation has to be identified and corrective actions should be taken. It is clear from the above, that having accurate models is critical. Unfortunately, creating vessel models is not an easy task. A vessel's performance sailing at sea is being affected by many parameters such as vessel design, speed, draft, trim, wind, waves, currents, sea water temperature, hull and propeller condition and many more. Because it is impossible to create a model taking into account all these parameters, engineers are trying to create simplified speed-power models (a model that will predict speed at a given power or vice versa), which account for some of the extra parameters and correct these models accordingly (i.e. ISO 15016).

Recently, two more methods of developing multivariate speed-power models gained popularity. One of them, Computational Fluid Dynamics (CFD), models the water flow and estimates drag coefficients to predict the total resistance to move a specific hull design through water. The alternative to CFD is to use the available data recorded from sensors and use advanced statistical methods (i.e. machine learning) to fit multivariate models to the available data. It is widely accepted that ISO 15016 generates accurate speed-power models. However speed trials are only conducted in very well controlled conditions (usually, but not always), in specific drafts and speed ranges, making it difficult to predict accurately performance outsides the boundaries of these ranges. In addition, it is known that procedures are not always followed correctly on speed trials resulting into speed-power models which are not accurate enough. CFD and machine learning techniques are gaining popularity as previously mentioned, however their credibility remains to be proven.

In this paper we apply a machine learning technique (Gaussian process regression) in an attempt to address three problems. Firstly, to check whether accurate speed-power models can be created when these are either not existing or not accurate. Secondly, to use a Bayesian approach to try to estimate the measured uncertainty and the uncertainty of the model. This is extremely useful as this uncertainty can be propagated at a second stage to relevant Key Performance Indicators (KPIs) and used to make more accurate decisions or prevent from making wrong decisions if the uncertainty is high. Finally, to check whether we can create an accurate enough fuel oil consumption model at a certain measured speed and draft and use this for measuring fuel consumption deviation over time.

## 2 Proposed probabilistic model

### 2.1 Overview

We propose a probabilistic model to predict the speed of a vessel based on measurements of power and mean draft. To model the vessel's speed, Gaussian process regression, *Williams et al. (2006)* is used:

$$\mathbf{s}(\mathbf{X}) \sim \mathcal{GP}\left(\mu_{s,\mathbf{X}}, \mathbf{K}_{\mathbf{X},\mathbf{X}}\right) + \boldsymbol{\varepsilon}$$

where:

$\mathbf{X}$ represents the measured input data and is composed of the two column vectors $\mathbf{p}$ and $\mathbf{d}$; i.e., $\mathbf{X} = \{\mathbf{p}, \mathbf{d}\}$. Thus, $\mathbf{X}$ has size $N \times 2$.

$N$ denotes the number of observed data points $\{s_i, p_i, d_i\}, i = 1, \dots, N$.

$\mathbf{p}$ is a column vector that contains the $N$ measurements $p_i$ of power delivered to the propeller.

$\mathbf{d}$ is a column vector that contains the $N$ measurements $d_i$ of the vessel's mean draft.

$\mathbf{s}$ is a column vector of $N$ Gaussian random variables that model the predicted speed of the vessel. More specifically, $\mathbf{s}$ predicts the measured speed of the vessel – and not the vessel's actual speed.

$\mathcal{GP}\left(\mu_{s,\mathbf{X}}, \mathbf{K}_{\mathbf{X},\mathbf{X}}\right)$ denotes a discretized Gaussian process with mean trend function $\mu_{s,\mathbf{X}}$, and covariance matrix $\mathbf{K}_{\mathbf{X},\mathbf{X}}$.

$\mu_{s,\mathbf{X}}$ is a mean trend function for the vessel's speed that defines the mean of $\mathcal{GP}\left(\mu_{s,\mathbf{X}}, \mathbf{K}_{\mathbf{X},\mathbf{X}}\right)$ before conditioning on $\mathbf{X}$.

$\mathbf{K}_{\mathbf{X},\mathbf{X}}$ is the covariance matrix of the Gaussian process.

$\boldsymbol{\varepsilon}$ is a column vector of size $N$. It is a noise term that accounts for both noise in the measurements and noise in the modelling error. The noise is modeled as a set of $N$ independent Normal random variables with zero mean and standard deviation $\sigma$.

### 2.2 Covariance kernel

The covariance matrix $\mathbf{K}_{\mathbf{X},\mathbf{X}}$ is obtained from the following isotropic covariance kernel with separable length scale:

$$k\left(p_i, p_j, d_i, d_j\right) = \eta^2 \cdot \exp\left[-\frac{\left(p_i - p_j\right)^2}{2 \cdot l_p^2}\right] \cdot \exp\left[-\frac{\left(d_i - d_j\right)^2}{2 \cdot l_d^2}\right]$$

where $i, j \in \{1, \dots, N\}$ and:

$\eta$ denotes the standard deviation associated with the covariance kernel.

$l_p$ denotes the correlation length that is used to model the correlation between $p_i$ and $p_j$.
$l_d$ denotes the correlation length that is used to model the correlation between $d_i$ and $d_j$.

### 2.3 Mean trend function

The mean trend function expresses the functional shape of how $s_i$ depends on $d_i$ and $p_i$ that one would expect based on physical consideration. In this case, the following functional relation is assumed:

$$\mu_{s_i} = \left(\frac{p_i}{\exp(c) \cdot d_i^w}\right)^{1/v}$$

where the mean trend function $\mu_{s,\mathbf{X}}$ is a column vector of all $N$ values of $\mu_{s_i}$. The quantities $v$, $w$ and $c$ are parameters whose values need to be determined during calibration of the model. The above functional relation is just one out of many potential function types. The performance with respect to other function types could be assessed in future studies.

### 2.4 Likelihood function

Given observed input measurements $\mathbf{X}$ and measured speeds $\tilde{\mathbf{s}}$, the likelihood of the observations can be evaluated as the density of a multivariate Normal distribution with mean vector $\mu_{s,\mathbf{X}}$ and covariance matrix $\mathbf{K}_{\mathbf{X},\mathbf{X}}$ evaluated at $\tilde{\mathbf{s}}$.

### 2.5 Sparse approximation of the Gaussian process

The performance of Gaussian process regression decreases with increasing number of observed data points. The computational complexity of Gaussian process regression is $O(M^3)$ and in terms of memory requirements it is $O(M^2)$, where $M = 2$ for the problem at hand. To use Gaussian process regression in combination with large datasets, sparse approximations can be employed.

In the sparse approximation to Gaussian process regression, $K$ inducing points are selected, where $K \ll M$. The inducing points are "strategically" placed in the input-domain. These points can be – but do not have to be – a subset of the original dataset. The inducing points can be chosen in advance or selected as part of the inference. The computational complexity of a sparse Gaussian approximation is typically $O(K \cdot M^2)$. However, as the full covariance matrix is not assembled, information in the data is compressed and high variance terms in the expansion are neglected. Therefore, the prediction of the Gaussian process is smoothened out.

In the context of this work, we applied the fully independent training conditional (FITC), *Quiñonero-Candela et al. (2005), Snelson et al. (2006),* approach to reduce the computational complexity of the problem at hand. In the FITC approach, the underlying assumption is that the training data is conditionally independent given the inducing points.

### 2.5 Uncertain model parameters and Bayesian model calibration

The values of following parameters are modelled as uncertain: $\eta$, $\sigma$, $l_d$, $l_w$, $c$, $v$, $w$. For each uncertain model parameter, a weakly informative prior distribution is selected that takes the physically and mathematically feasible parameter space into account. The product of the joint prior density function and the likelihood function is proportional to the posterior distribution. The values of the uncertain model parameters are learned based on recorded measurements $\{s_i, p_i, d_i\}, i = 1, \ldots, N$ of a specific vessel. We selected the values for the uncertain model parameters such that the density of the posterior distribution is maximized through application of an optimization algorithm. The uncertain model parameters were fixed to the so-obtained values, which corresponds to the maximum a posteriori probability (MAP) approach.

### 2.6 Using the model for prediction

Using the above described sparse Gaussian process regression, for each value of power $p_*$ and draft $d_*$, the mean $\mu_*$ and standard deviation $\sigma_*$ of the predicted speed measurement can be obtained from a conditional multivariate Normal distribution. The covariance between different predicted speeds is not explicitly considered in the context of this work.

## 3. Application of probabilistic model in theoretical speed

### 3.1 Mean trend of the probabilistic model versus speed trials (prior distribution)

As previously discussed, the reason for applying the proposed probabilistic model is, firstly, to replicate or correct (if possible) speed-power curves in case they do not exist and, secondly, to estimate the uncertainty of the model and the uncertainty due to noise. In order to validate the model a 14000 TEU container vessel was selected. This vessel is equipped with automatic datalogger acquiring data averaged every 15 minutes and historical data for the last 4 years were made available. Among logged parameters are speed through water, shaft power, fuel consumption, draft aft and fore, wind speed and wind direction, speed over ground and rpm. In order to minimize the effects of possible hull performance deterioration, the first year of data was used as a training set after applied filtering and data cleaning processes. In order to account for calm sea state, true wind values higher than 16 kn were filtered out. The computed coefficients after fitting the model can be seen in Table I.

Table I: Computed coefficients for speed-draft-fuel consumption relationship

| c | w | v |
|---|---|---|
| -0.78 | 3.06 | 0.71 |

In Fig.1, the measured data are plotted together with the speed-power curves that are predicted from the mean trend of the probabilistic model and compared against the speed-power curves from speed trials. It is obvious that the curve from speed trials in laden conditions is not correct. The two different model curves in ballast conditions appear to be closer in absolute terms and it is difficult to conclude on which one is more correct. When one is interested in monitoring “absolute performance”, although not possible as such, *Paereli et al. (2017)*, the position of the curve is very important and in this case, the mean trend of the probabilistic model appears to fit the data better (at least in laden conditions). However, when measuring relative performance, as in ISO19030, the position of the curve is not that important compared to the curve’s shape.

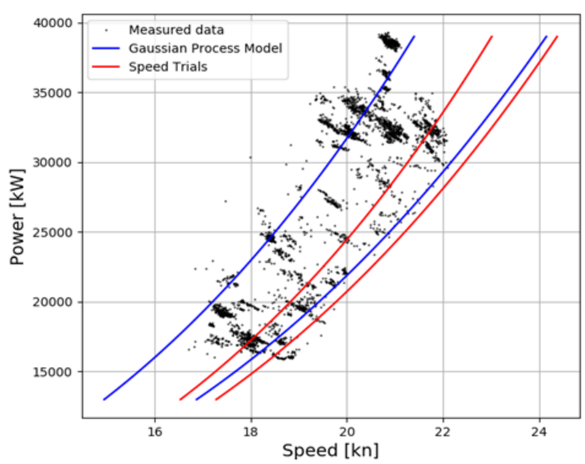


Fig.1: Measured values vs mean trend of probabilistic model and speed trials model

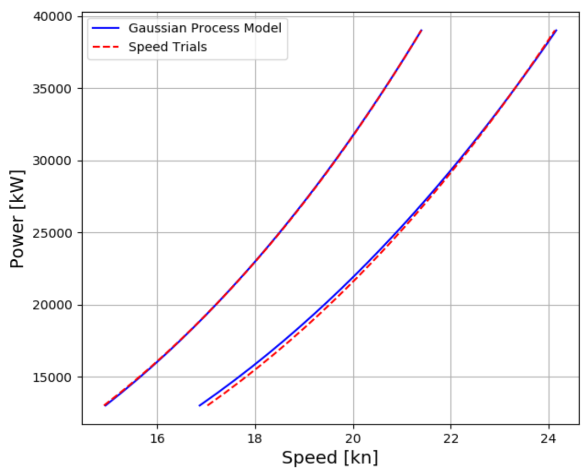


Fig.2: Comparison of curves’ shape between the two models

In Fig.2, the speed trial curves are shifted accordingly in order to compare the shape of the curves. The curves in laden conditions between the two models are almost identical. However, comparison of two curves in ballast conditions, shows differences, especially in lower speed ranges. The differences observed in low ranges (~0.2 kn) might be considered significant, however, when speed trials results are not available the mean trend of the probabilistic model can be used as a speed-power curves model in order to quantify changes in hull and propeller performance.

### 3.2 Maximum a posteriori probability (MAP)

In the previous chapter, the mean trend of the proposed probabilistic model was discussed and evaluated. However, the method applied in this paper does not only predict the mean trend of the theoretical (optimal) speed, but also quantifies the uncertainty in the prediction given the data. Using the sparse Gaussian process regression (described in chapter 2), for each value of power and draft, the mean $\mu_*$ and standard deviation $\sigma_*$ of the predicted speed measurement can be obtained from a conditional multivariate Normal distribution. In order to evaluate the output of the model, the same dataset as in the previous chapter was used.

Fig.3 shows the prediction (posterior belief) of the model for a certain draft (16.1 m). In this speed-power plot, the observed data for a draft range of 0.6 m was plotted (15.8 m - 16.4 m). It is obvious that the prediction is influenced (corrected) based on the existence of data at the draft in question (16.1 m). This is exactly as expected by the Gaussian process regression model. In addition, the two standard deviation intervals are plotted. In ranges better described by data (17.0 kn - 18.0 kn and 19.5 kn - 20.5 kn) the two standard deviation intervals narrow significantly. This means that the uncertainty decreases because there is data in these specific ranges. Finally, in ranges where we don't have enough data the prediction is similar to the mean trend. This is expected as we have no data (evidence) to improve our prior belief.

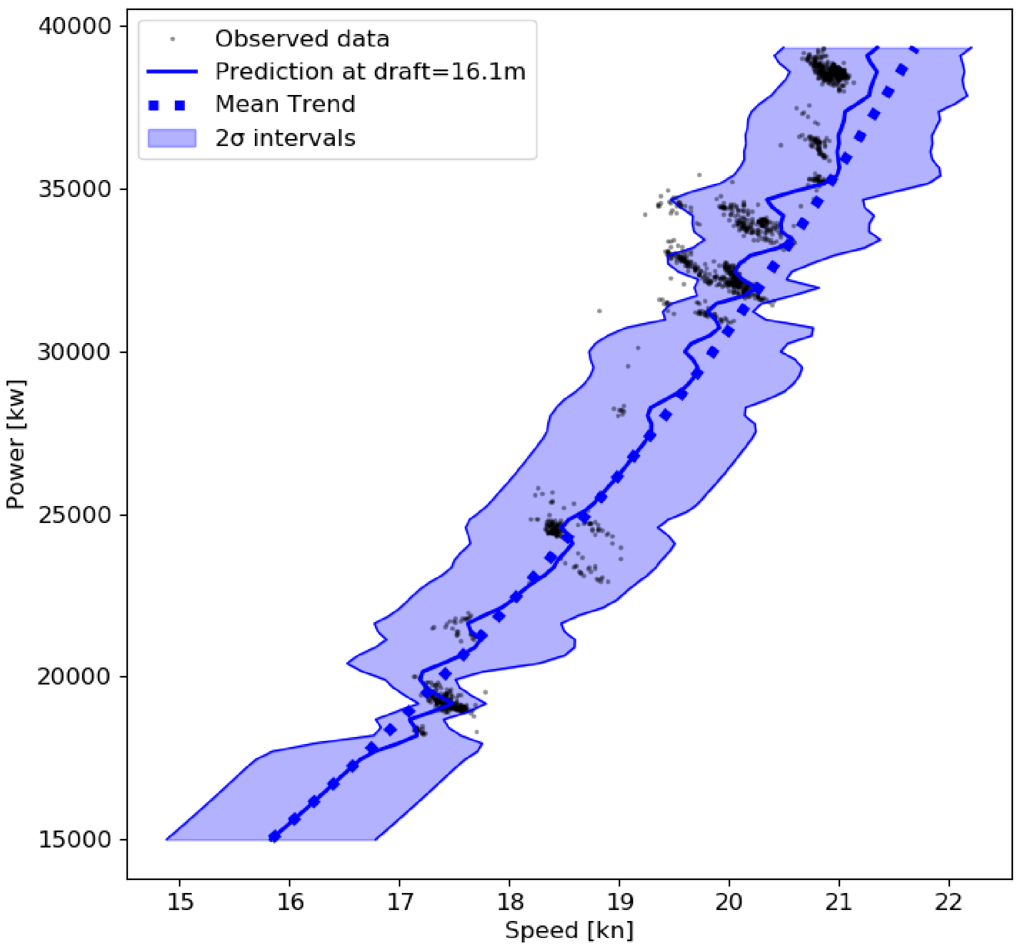


Fig.3: Observed data compared to mean trend and uncertainty bounds of the prediction

In order to evaluate the accuracy of the model and conclude whether this model can be used instead of speed trials model the first year of the available dataset was split into training (75%) and test set (25%). The model was fitted to the training set and the results were compared against the test set. In Fig.4, it can be observed that for most of the test period (with some very small exceptions), predicted speed values are close to the logged values. The standard deviation of the relative error distribution is 2.3%, which in absolute terms means 0.4 kn.

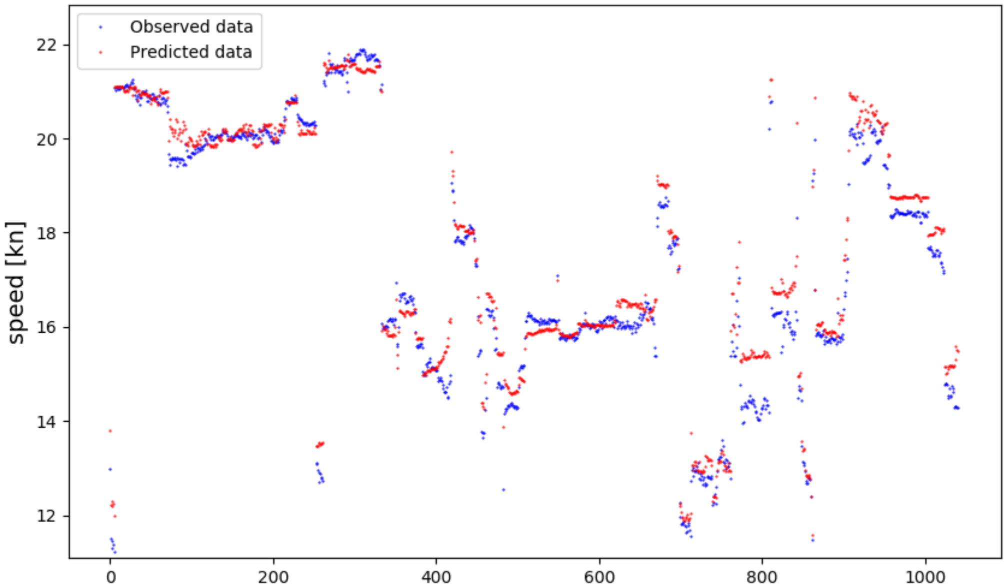


Fig.4: Logged and predicted speed through water (test period)

To conclude whether the proposed probabilistic model can be generally used and not only providing good results in specific cases, same approach as described above has been followed in other three cases (two other containerships and one VLCC). In all cases only the first year of data after dry-docking has been used, 75% of it being the training dataset and 25% - test dataset. In all three cases, predicted speed was found to match very well the logged values.

### 3.3 Uncertainty

As discussed in the introduction, the purpose of this paper is also to try to estimate the uncertainty of the model and the uncertainty due to noise and apply it in KPIs. In this chapter, the calculation of uncertainty is applied in one of the ISO19030 KPIs (in-service performance) and the results are discussed.

The speed deviation (as per ISO19030) at the $i$th measurement data-point is:

$$v_{d,i} = \frac{\widetilde{s_i}}{s_i} \text{-} 1$$

where $\widetilde{s_i}$ is the speed measured at the respective point, and $s_i$ is the predicted speed for this data point. A first-order approximation for the variance of $v_{d,i}$ is:

$$Var[V_{d,i}] \approx \left(\frac{\widetilde{s_i}}{s_i^2}\right)^2 \cdot Var[s_i^2]$$

Variance of the predicted speed is already provided by the model described here. The same dataset from the same 14000 TEU container was used. A period of 1 month was isolated from the second year of the provided dataset. In Fig.5, the values are plotted together with the error bars at two standard deviations.

For visualization purposes, error bars are plotted every 10 points. The calculated variance from the model can be used in order to calculate the standard error of the mean (SEM). In our example the SEM is 0.05 kn. The 95% confidence interval is given by ±2 x SEM and in our case ± 0.1 kn.

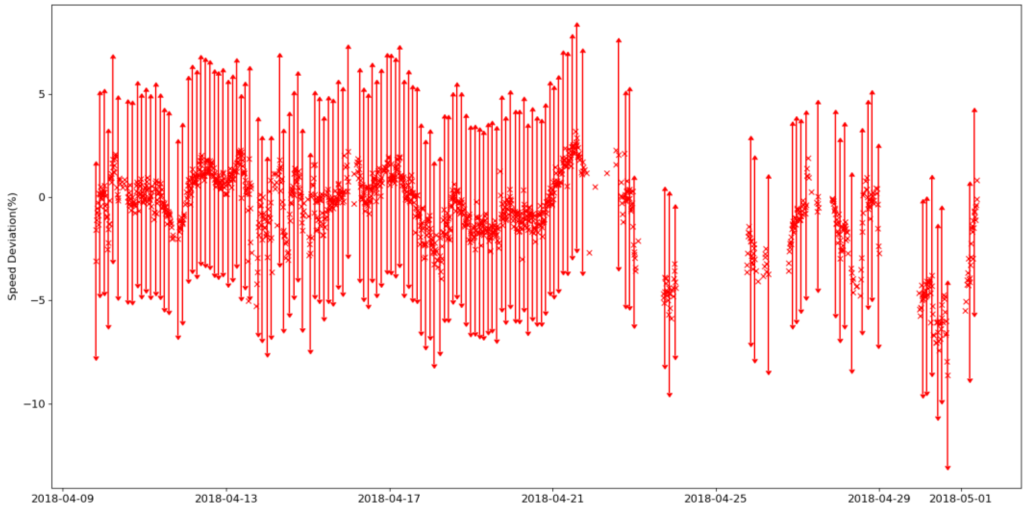


Fig.5: Mean and standard deviation error bars of mean percent speed deviation

## 4. Further application of probabilistic modeling

As discussed in the previous chapter, probabilistic modeling appears to give good results when predicting vessel's speed through water for given (measured) delivered power and mean draft. Having a trustful model at hand is very useful, especially when speed-power reference curves are not available. This makes it possible calculating speed loss – the performance value mentioned in ISO19030. This standard has gain popularity in the last 3 years and gradually more people can now benefit from using its guidelines towards quantifying the changes in time in their vessels' hull and propeller performance. There are, however, people who are more inclined towards computing a performance indicator based on fuel consumption and there is no wonder why. Being able to compute some sort of fuel consumption deviation, bunker buyers would be very happy to learn directly about their costs increase. Unfortunately, not that many different attempts have been made to somehow quantify changes in vessel fuel consumption in time. ISO19030 supporters usually like the easy, but nevertheless commonly accepted in the industry 1:3 rule of thumb and go down this road. Another big group of people probably try building speed-fuel consumption reference curves by different means. Generally speaking, one can try generating speed-fuel consumption reference curves from logged data by either relying on statistical modeling or probabilistic modeling. Since probabilistic modeling approach by means of Gaussian process regression has been found useful in predicting vessel speed through water, it is decided to further apply it in fuel consumption prediction.

A very similar approach to the one described in the previous chapter has been used for predicting fuel consumption values. A Gaussian process regression model which describes the relationship between vessel speed through water and draft mean is proposed below.

$$FOC = e^c \cdot V_S^v \cdot D^w$$

The proposed model has been built and applied on a dataset from VLCC. This vessel is equipped with automatic datalogger acquiring data every 15 minutes and historical data for the first year since last dry-docking has been made available. Among logged parameters are speed through water, shaft power, fuel consumption, draft aft and fore, wind speed and wind direction, speed over ground and rpm.

Similar to the case when predicting speed through water, the model was built on a training dataset – in this case the first half a year with data (the first half of the reference period in ISO19030 terms). Data has been cleaned for outliers and validated as per ISO19030. Furthermore, data has been filtered for true wind speed below 16 kn, speed above 7.5 kn and fuel consumption values below 120 T/d. The relationship between speed and fuel consumption is shown in Figs.6 and 7.

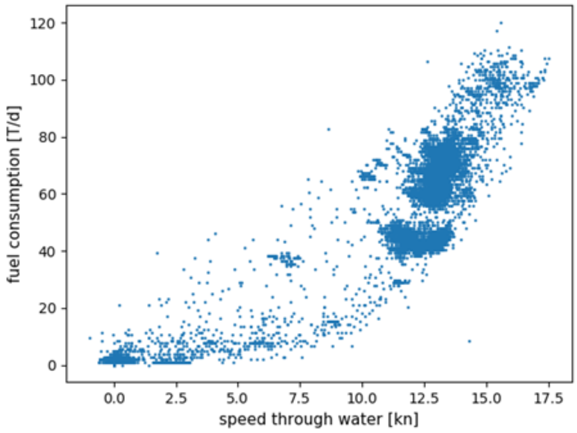


Fig.6: Speed through water - fuel consumption relationship (first half of Year 1)

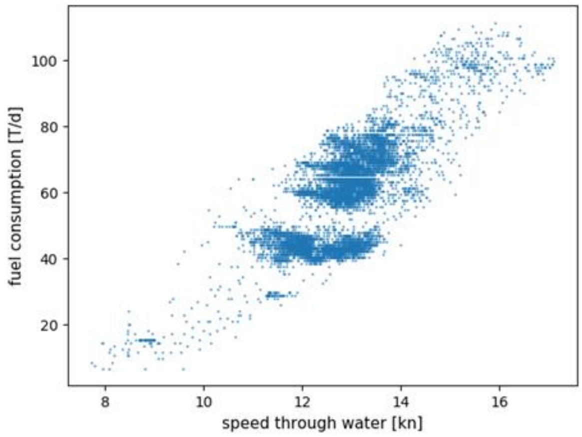


Fig.7: As Fig.6, but cleaned

Upon applying Gaussian process regression on the isolated and cleaned, in a similar way as described above, training dataset, output parameters of the model have been generated, Table II.

Table II: Computed coefficients for speed-draft-fuel consumption relationship

| $c$ | $v$ | $w$ |
|---|---|---|
| -3.66 | 2.65 | 0.31 |

Once the model is built, ideally, one would compare with other sources such as model tests or speed trials. In this case, however, no other models in terms of speed through water and fuel consumption are available. Therefore, in order to evaluate how good the model is, i.e. what the model uncertainty is, in predicting fuel consumption values, one would need to apply this model on a test dataset. As test dataset the second half of the first year with data has been chosen. The assumption made here is that there is no significant (quantifiable) change in vessel performance in the first year after dry-docking, something that has been documented both visually by inspecting the vessel and by analyzing changes in hull and propeller performance according to ISO19030-2. Since the relationship between speed through water and fuel consumption is not expected to have changed throughout the first year, the model is considered good for use in further vessel performance analysis if predicted fuel consumption values do not differ much from actual logged values. Figs.8 and 9 show results confirming this.

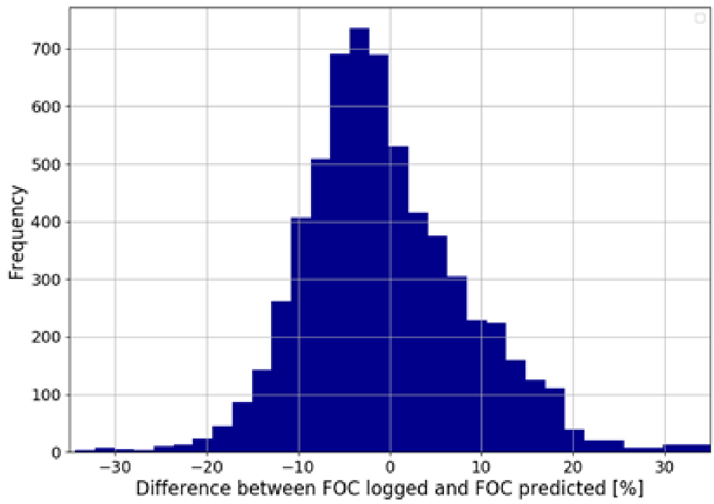


Fig.8: Relative error in FOC prediction

In Fig.9, predicted fuel consumption and logged fuel consumption values are plotted over both training and test periods. In the training period, relative prediction error varies between -2% and 2%. Such a small error is not surprising since this is the data that has been used for training and building the model. It is, however, interesting to see how well the model works on the test dataset. For most of the period (with some very small exceptions), predicted FOC values are close to the logged values. The standard deviation of the relative error distribution in Fig.8 is 10%, which in absolute terms means 5.8 T/d. These relatively small prediction errors, but also slightly larger ones during some short periods, are linked to uncertainties in speed, draft and fuel consumption measurements, but also weather and sea state. To visualize this, the model has been applied to the test dataset which is not filtered for true wind speed. Results are shown in Fig.10.

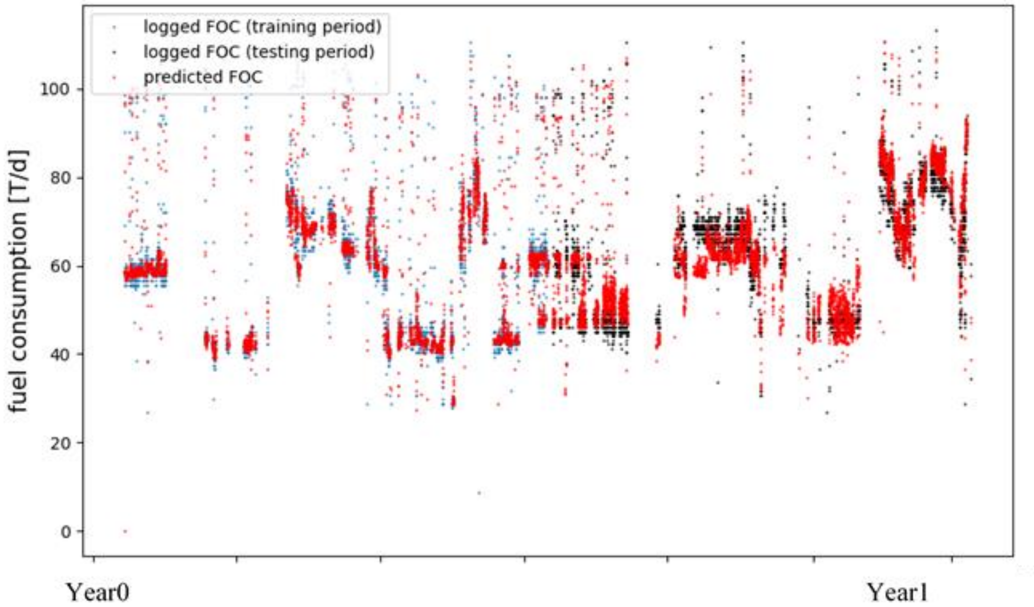


Fig.9: Logged and predicted FOC

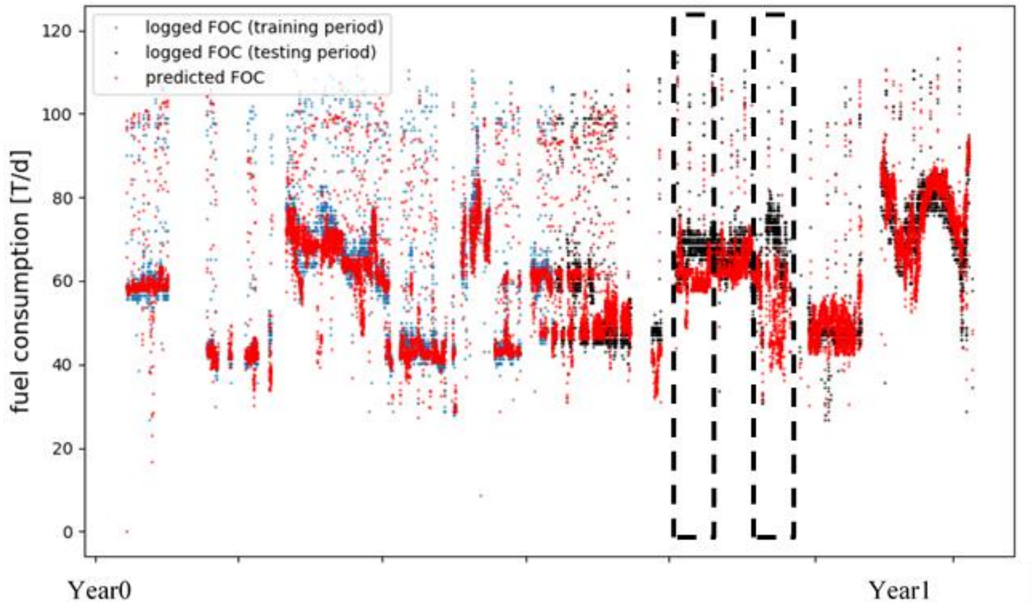


Fig.10: Logged and predicted FOC

The observation to be made in Fig.10 is that during the periods with true wind speed higher than 16 kn in the test dataset (shown with dotted lines), the prediction error increases, just as expected. This is, one the one hand side, because the logged fuel consumption is higher in those periods for the same draft condition and speed through water, and, on the other hand side, because predicted fuel consumption is the “ideal” one – the one computed for conditions determined by the filter used for isolating the training dataset.

The standard deviation of the relative error in periods marked with dotted lines in figure 10 is 16% which in absolute terms means 7.6 T/d. Seeing such differences between logged fuel consumption and predicted (“ideal”) fuel consumption in periods where vessel performance could be concluded stable, for example by running analysis according to ISO19030-2, brings a follow-up idea. One could use this difference to learn about the impact of wind on the relationship between speed through water and fuel consumption. Alternatively, one could try to implement a model in which fuel consumption is a function of not only vessel speed and loading condition, but also function of true wind speed and true wind direction. This is outside of the scope of this paper, but work in this direction will continue.

## 5. Conclusion

Gaussian process regression was applied to in-service data from several ships. This Bayesian approach helped modeling the vessel specific relationship between speed through water and shaft power. A mean trend function describing the model was proposed, and the latter was fitted to dataset (the first year after dry-docking) from four different vessels. Upon computing the coefficients of the model, this was compared with vessel model (either from model tests or speed trials) provided by vessel owner. In all cases it was concluded that the shape of the computed model is quite similar to the shape of the given model and if the latter had not been made available, computed model could have very well been applied to quantify changes in hull and propeller performance in time.

Furthermore, the applied technique appeared useful in estimating the measured uncertainty and the uncertainty of the model. For evaluating the accuracy of the model, the first year with data after dry-docking was split into a training dataset – to which the model was fit, and a test dataset – on which the model was tested. The chosen split ratio was 75%/25%. Upon applying the built model on the test dataset relatively small difference have been concluded between the predicted values and logged speed through water values. If small prediction error solely represents model uncertainty and not partly also speed through water/draft/shaft power uncertainty, then it could be concluded that in the very end such a model uncertainty results in a standard error of the mean of speed deviation of ±0.05 kn.

Finally, Gaussian process regression has been applied to in-service data from a VLCC for predicting fuel consumption. First, an equation of the model was suggested and then the latter was fitted to a training dataset (the first half a year after vessel dry-docking). After that the built model was verified on a test dataset (the second half of the first year after dry-docking). Predicted fuel consumption values were concluded close to the logged values. Standard deviation of the relative error distribution was 10%, which in absolute terms was 5.8 T/d.

Gaussian process regression was found to be useful in building vessel specific speed-power-draft and speed-fuel consumption-draft models. This approach can, one the one hand side, solve the problem of not having a vessel model available from ship owner, on the other hand side, estimate the uncertainty of the final performance values, and, last but not least, eventually, enable estimation of the effect of other parameters (e.g. true wind speed, true wind direction, etc.) on the performance values.

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